

Fórmulas de derivación numérica

1. Fórmulas de diferencias finitas divididas hacia adelante

1.1. Primera derivada

$$f^I(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h) \quad (1)$$

$$f^I(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h} + O(h^2) \quad (2)$$

1.2. Segunda derivada

$$f^{II}(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O(h) \quad (3)$$

$$f^{II}(x_i) = \frac{-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)}{h^2} + O(h^2) \quad (4)$$

1.3. Tercera derivada

$$f^{III}(x_i) = \frac{f(x_{i+3}) - 3f(x_{i+2}) + 3f(x_{i+1}) - f(x_i)}{h^3} + O(h) \quad (5)$$

$$f^{III}(x_i) = \frac{-3f(x_{i+4}) + 14f(x_{i+3}) - 24f(x_{i+2}) + 18f(x_{i+1}) - 5f(x_i)}{2h^3} + O(h^2) \quad (6)$$

1.4. Cuarta derivada

$$f^{IV}(x_i) = \frac{f(x_{i+4}) - 4f(x_{i+3}) + 6f(x_{i+2}) - 4f(x_{i+1}) + f(x_i)}{h^4} + O(h) \quad (7)$$

$$f^{IV}(x_i) = \frac{-2f(x_{i+5}) + 11f(x_{i+4}) - 24f(x_{i+3}) + 26f(x_{i+2}) - 14f(x_{i+1}) + 3f(x_i)}{h^4} + O(h^2) \quad (8)$$

2. Fórmulas de diferencias finitas divididas hacia atrás

2.1. Primera derivada

$$f^I(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} + O(h) \quad (9)$$

$$f^I(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h} + O(h^2) \quad (10)$$

2.2. Segunda derivada

$$f^{II}(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2} + O(h) \quad (11)$$

$$f^{II}(x_i) = \frac{2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2}) - f(x_{i-3})}{h^2} + O(h^2) \quad (12)$$

2.3. Tercera derivada

$$f^{III}(x_i) = \frac{f(x_i) - 3f(x_{i-1}) + 3f(x_{i-2}) - f(x_{i-3})}{h^3} + O(h) \quad (13)$$

$$f^{III}(x_i) = \frac{5f(x_i) - 18f(x_{i-1}) + 24f(x_{i-2}) - 14f(x_{i-3}) + 3f(x_{i-4})}{2h^3} + O(h^2) \quad (14)$$

2.4. Cuarta derivada

$$f^{IV}(x_i) = \frac{f(x_i) - 4f(x_{i-1}) + 6f(x_{i-2}) - 4f(x_{i-3}) + f(x_{i-4})}{h^4} + O(h) \quad (15)$$

$$f^{IV}(x_i) = \frac{3f(x_i) - 14f(x_{i-1}) + 26f(x_{i-2}) - 24f(x_{i-3}) + 11f(x_{i-4}) - 2f(x_{i-5})}{h^4} + O(h^2) \quad (16)$$

3. Fórmulas de diferencias finitas divididas centradas

3.1. Primera derivada

$$f^I(x_i) = \frac{f(x_{i+1}) - f(x_{i-1}))}{2h} + O(h^2) \quad (17)$$

$$f^I(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2}))}{12h} + O(h^4) \quad (18)$$

3.2. Segunda derivada

$$f^{II}(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1}))}{h^2} + O(h^2) \quad (19)$$

$$f^{II}(x_i) = \frac{-f(x_{i+2}) + 16f(x_{i+1}) - 30f(x_i) + 16f(x_{i-1}) - f(x_{i-2}))}{12h^2} + O(h^4) \quad (20)$$

3.3. Tercera derivada

$$f^{III}(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + 2f(x_{i-1}) - f(x_{i-2}))}{2h^3} + O(h^2) \quad (21)$$

$$f^{III}(x_i) = \frac{-f(x_{i+3}) + 8f(x_{i+2}) - 13f(x_{i+1}) + 13f(x_{i-1}) - 8f(x_{i-2}) + f(x_{i-3}))}{8h^3} + O(h^4) \quad (22)$$

3.4. Cuarta derivada

$$f^{IV}(x_i) = \frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2}))}{h^4} + O(h^2) \quad (23)$$

$$f^{IV}(x_i) = \frac{-f(x_{i+3}) + 12f(x_{i+2}) - 39f(x_{i+1}) + 56f(x_i) - 39f(x_{i-1}) + 12f(x_{i-2}) - f(x_{i-3}))}{6h^4} + O(h^4) \quad (24)$$